Literature

- Westerståhl, Generalized Quantifiers, SEP.
- Peters & Westerståhl, Quantifiers in Language and Logic, OUP, 2008.
- Handbook of Logic and Language, Van Benthem & Ter Meulen (Eds.), Elsevier 2011.
- Szymanik, Quantifiers & Cognition, Studies in Linguistics & Philosophy, Springer, 2016.

Determiners: Examples

(1) **All** poets have low self-esteem.

(2) **Some** dean danced nude on the table.

(3) **At least 3** grad students prepared presentations.

(4) **An even number** of the students saw a ghost.

(5) **Most** of the students think they are smart.

(6) **Less than half** of the students received good marks.

(7) **Many** of the soldiers have not eaten for **several** days.

(8) **A few** of the conservatives complained about taxes.

And many more. . .

Determiners

Definition

Expressions that appear to be descriptions of quantity.

Example

All, not quite all, nearly all, an awful lot, a lot, a comfortable majority, most, many, more than *n*, less than *n*, quite a few, quite a lot, several, not a lot, not many, only a few, few, a few, hardly any, one, two, three.

Quantifiers are second-order relations

Observation

If we fix a model $\mathbb{M} = (M, A^M, B^M)$, then we can treat a generalized quantifier as a relation between relations over the universe.

Example

$$\text{every}[A, B] = 1 \text{ iff } A^M \subseteq B^M$$

$$\text{even}[A, B] = 1 \text{ iff } \text{card}(A^M \cap B^M) \text{ is even}$$

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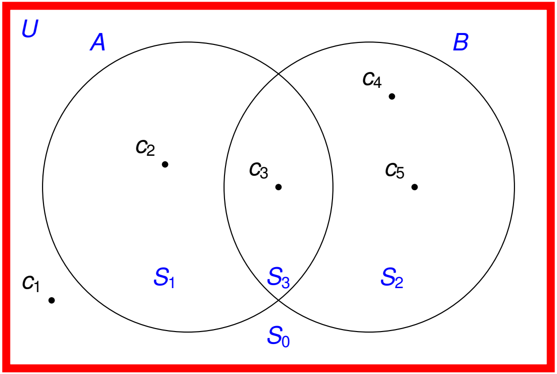
Illustration



Generalized Quantifiers

Definition

A quantifier Q is a way of associating with each set M a function from pairs of subsets of M into $\{0, 1\}$ (False, True).

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Space of GQs

Q is a function from M into a function from pairs of subsets of M into $\{0, 1\}$.

- If $\text{card}(M) = n$, then there are $2^{2^{2n}}$ GQs.
- For $n = 2$ it gives 65,536 possibilities.

Question

Which of those are realized in natural language as determiners?

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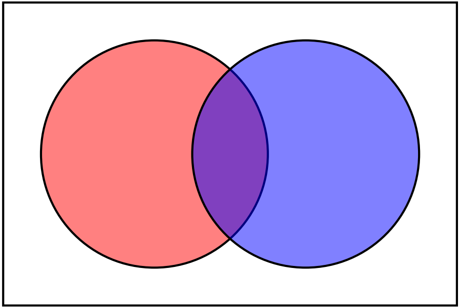
Outline

Isomorphism closure

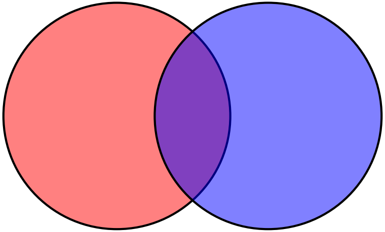
(ISOM) If $(M, A, B) \cong (M', A', B')$, then $Q_{\mathbf{M}}(A, B) \Leftrightarrow Q_{\mathbf{M'}}(A', B')$



Topic neutrality

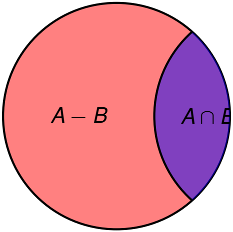
Extensionality

(EXT) If $M \subseteq M'$, then $Q_{\mathbf{M}}(A, B) \Leftrightarrow Q_{\mathbf{M'}}(A, B)$



Conservativity

(CONS) $Q_{\mathbf{M}}(A, B) \Leftrightarrow Q_{\mathbf{M}}(A, A \cap B)$



Monotonicity

$\uparrow$MON $Q_M[A, B]$ and $A \subseteq A' \subseteq M$ then $Q_M[A', B]$.

$\downarrow$MON $Q_M[A, B]$ and $A' \subseteq A \subseteq M$ then $Q_M[A', B]$.

MON$\uparrow$ $Q_M[A, B]$ and $B \subseteq B' \subseteq M$ then $Q_M[A, B']$.

MON$\downarrow$ $Q_M[A, B]$ and $B' \subseteq B \subseteq M$ then $Q_M[A, B']$.

Inference test

(1) Some boy is dirty.
(2) Some child is dirty.

(1) All children are dirty.
(2) All boys are dirty.

(1) All boys are muddy.
(2) All boys are dirty.

(1) No boy is dirty.
(2) No boy is muddy.

(1) Exactly five children are dirty.
(2) Exactly five boys are dirty.

The study of such ‘easy’ inferences on surface forms goes by the name ‘natural logic’ which is a thriving area of research

Monotonicity Universal

Monotonicity Universal (Barwise & Cooper 1981)

All simple determiners are monotone or conjunctions of monotone determiners

Research questions

- Do all NL determiners satisfy ISOM, EXT, MON, and CONS?
- Only? Every third? An even number of?
- Do all **simple** NL determiners satisfy ISOM, EXT and CONS?